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Endogenous and exogenous commitment

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HIGHLIGHTS

- I study a repeated version of Hart and Tirole's (1988) dynamic monopoly game.
- I provide a folk theorem for the payoffs of the long-run monopolist.
- I characterize the ability to commit under high-frequency contracting.
- More exogenous commitment can make endogenous commitment more difficult.

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ABSTRACT

I study a dynamic model of monopoly sales in which one long-term monopolist without exogenous commitment power interacts with a sequence of short-term consumers with private valuations. I provide a folk theorem which characterizes the payoffs of a sufficiently patient monopolist and I show that exogenous commitment power can obstruct the endogenous commitment from repeated game incentives.

1. Introduction

The ability to commit is important in mechanism design and contract theory. Generally, not having full commitment is costly. A large literature has arisen around how contracts would be written when the mechanism designer does not have full commitment power. However, many of the "principals" we see in real life act in a repeated setting, which may open opportunities for endogenous commitment. This paper studies how the exogenous commitment of a legal system interacts with endogenous commitment from the repeated game.

As the stage game, I use the model of Hart and Tirole (1988) in which a monopolist seller interacts with a buyer who has private information about his valuation for a nondurable consumption good. I assume that the seller faces a sequence of finitely lived buyers and compare the equilibria of the repeated game under full commitment, spot contracting, and renegotiation. Previous work has noted that reputation in a repeated game could overcome the principal's inability to commit but has not shown how the "exogenous" commitment power which is provided by the modeler or legal system interacts with the "endogenous" commitment power that arises from repeated game incentives.

2. Model

A monopolist seller is infinitely lived and faces a sequence of buyers, each of which interacts with the seller for $T < \infty$ periods, with only a single buyer active at any point. I refer to the *stage game* as the strategic interaction between the seller and a single buyer, while the *repeated game* refers to the full sequence of strategic interactions (Fudenberg et al., 1990).

In each period, the seller can produce a perishable consumption good at a normalized cost of 0. Each buyer i demands a single unit of the consumption good in each period and has a valuation

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$b_i \in \{\underline{b}, \bar{b}\}$ where $0 < \underline{b} < \bar{b}$. This value in known to the buyer but unknown to the seller. Each buyer's value is independently and identically distributed, with $\mathbb{P}(b_i = \bar{b}) = \mu$. It is well known that in the presence of full commitment, the monopolist sells only to high types at price $\bar{b}$ in all periods if $\mu\bar{b} > \underline{b}$ and sells to everyone at price $\underline{b}$ if $\mu\bar{b} < \underline{b}$ (Hart and Tirole, 1988). I will focus on the case in which $\mu\bar{b} > \underline{b}$, as the full commitment outcome is attainable without commitment when this inequality does not hold.

The seller's strategy space in each round depends on the exogenously provided commitment structure. In addition to full commitment, the two cases I consider are *spot contracting* and *commitment with renegotiation*. In the spot contracting game, the seller can only commit to consumption and price in the current period. In the commitment with renegotiation game, the seller can commit to a sequence of prices and consumptions for the future, but cannot commit to not renegotiate this offer.

The buyer's strategy space depends on the choices of the seller. If no new contract offer is made in a period, then the buyer has no choice to make. If the seller offers a new contract, the buyer can choose whether to accept or reject it.

Both players discount at rate $\delta < 1$. Payoffs for the seller are the discounted sum of revenue. Payoffs for the buyer are the discounted utility from consumption minus any transfers. In what follows, these payoffs will be normalized by $1 - \delta$.

I focus on Perfect Bayesian Equilibria, which in this setting is defined as follows. The seller's and each buyer's actions are sequentially optimal, which is to say that in any period given their beliefs about the other player's strategy and (in the case of the seller) the other player's type, they are maximizing their expected payoffs. The seller's beliefs about the valuation of the buyer only change when the buyer takes an action, and are required to satisfy Bayes' rule whenever possible.

3. Results

The approach to studying equilibria of the repeated game is mostly standard, in that it focuses on characterizing the worst continuation equilibrium the long-run player can face, which in this situation is the equilibrium of the stage game. The only major difference in this model is that the stage game itself is dynamic, and discount rates affect the payoffs which arise in the stage game. The first result characterizes the payoffs the seller can receive if she is sufficiently patient. The payoffs for the case in which $\delta = 1$ were first computed by Schmidt (1993), and I will restate them here.[2] Define

$$v = \bar{b} - \frac{1}{T}(1-\mu)\bar{b}\sum_{t=1}^{T}\left(\frac{\bar{b}}{\bar{b}-\underline{b}}\right)^{t-1}$$

if $\exists n \geq T$ s.t. $1 - \left(\frac{\bar{b}-\underline{b}}{\bar{b}}\right)^{n} \leq \mu < 1 - \left(\frac{\bar{b}-\underline{b}}{\bar{b}}\right)^{n+1}$, and

$$v = \frac{T-n}{T}\underline{b} + \frac{n}{T}\bar{b} - \frac{1}{T}(1-\mu)\bar{b}\sum_{t=1}^{n}\left(\frac{\bar{b}}{\bar{b}-\underline{b}}\right)^{t-1}$$

if $n < T$. The value v is the payoff that a monopolist receives when optimally screening buyers in periods $T - n$ to $T - 1$.

Proposition 1. *In both the spot contracting and the renegotiation settings and for any $\pi \in [v, \mu\bar{b}]$, for δ high enough there exists an equilibrium of the repeated game in which the seller receives π.*

Proof. See Appendix.

This result is the "folk theorem" for the seller's payoffs. The upper bound on her equilibrium payoffs is what she would receive in the full commitment game, while the lower bound is what she receives in the equilibrium of the stage game when δ gets close to 1.

In this paper, T describes the number of periods in which the buyer and seller will interact rather than the *length* of the contract (which is fixed). This interpretation suggests studying not the discount rate between periods within a buyer's "life", but rather the seller's discount factor between waves of buyers. Thus, I will define $\beta = \delta^T$ and study how the set of feasible equilibria varies when δ and T change, but β is held constant.

A natural question is under for what values of β the seller is able to attain the payoffs that she would receive if she had full commitment exogenously provided. The seller benefits from deviating by selling to a low type when she would otherwise be earning 0. The worst punishment that the seller can receive for deviation is a reversion to the stage game equilibrium. However, the discount rate affects both the weight of payoffs from the future and the *form* of the punishment equilibrium. Thus, it is not obvious that there exists a unique β which equates the full commitment payoffs to the discounted payoffs from deviating, then entering the punishment equilibrium. Despite this fact, I am able to make a few observations about the set of β for the spot contracting and renegotiation settings.

Proposition 2. *If contracts are offered and renegotiated quickly enough ($T \to \infty$), the full commitment payoffs are attainable in both the spot contracting and renegotiation settings if*

$$\beta > \frac{\underline{b}}{\mu\bar{b}}$$

Proof. The monopolist has the strongest incentive to deviate in the second period of interaction with a low valuation buyer, so the inequality that δ must satisfy is

$$\delta^{T-1}\mu\bar{b} \geq \frac{1-\delta^{T-1}}{1-\delta}\underline{b} + \delta^{T-1}((1-r(T))\underline{b} + r(T)v(T)),$$

where $r(T)$ is the proportion of periods in which the monopolist sets a price higher than $\underline{b}$ and $v(T)$ is the discounted average profits earned by the monopolist in the screening periods, which must be less than $\bar{b}$. Propositions 2 and 3 of Hart and Tirole (1988) show that as $T \to \infty$, $r(T) \to 0$. Furthermore, $\delta^{T-1} \to \beta$, giving the result. □

Proposition 2 states that when contract offers and renegotiations happen with high frequency, all sellers whose discount factor is above a particular cutoff will be able to commit to the full commitment allocation in the repeated game. The cutoff is the same in both the spot contracting and the commitment with renegotiation settings because for large T, the seller can only extract extra surplus from the high types in a few periods, so the discounted per-period payoffs converge to $\underline{b}$.

It is unlikely that the cutoffs under high frequency contract offers in the two commitment settings will be equal to each other with different cost or demand structures. The intuition behind the form of contracting when T is large is qualitatively different between the two settings. When the monopolist can commit with renegotiation, contracts must converge to the efficient contract early in the relationship, making it impossible for the seller to extract additional surplus by screening with inefficient contracts. In the spot contracting setting, the seller pools buyers for most of the relationship, putting off any learning. These lead to equivalent payoffs with unit demand and binary types because the pooling

[2] Schmidt (1993) describes a model in which a *seller* has private information about its *costs* and the buyer makes offers. However, when there are only two possible costs, their model can be shown to be equivalent to the one found here. In particular, after accounting for notational differences, the value v is equivalent to the Buyer's payoffs given in Schmidt's Proposition 3.

allocation is equivalent to the efficient allocation, but this need not be the case for more general payoff structures.

I conclude with a result about how the ability of a seller to commit varies with the exogenous level of commitment.

Theorem 1. *There is a collection of parameters μ, $\underline{b}$, $\bar{b}$, T, and β for which commitment payoffs are attainable in the spot contracting setting but not the renegotiation setting.*

Proof. When implementing the commitment equilibrium, the seller earns the same payoffs in both settings. Furthermore, the optimal deviation in both settings (starting to sell to the low valuation type in the second period) gives the same payoffs in both settings. Thus, any differences in the ability to attain commitment payoffs arise as a result of different deviation payoffs. These deviation payoffs are the payoffs from the stage game.

As noted in the proof of Proposition 1, the δ necessary to make commitment payoffs attainable in equilibrium approaches 1 as T gets arbitrarily large. Thus, in the punishment equilibrium of the spot contracting game, if

$$1-\left(\frac{\bar{b}-\underline{b}}{\bar{b}}\right)^{n}\leq\mu<1-\left(\frac{\bar{b}-\underline{b}}{\bar{b}}\right)^{n+1},$$

then the monopolist is selling only to high types in periods $T-n$ to $T-1$. Take the lowest value of δ such that the commitment payoffs are attainable. Because T can be chosen high enough to make δ arbitrarily close to 1, the sequence of prices and probabilities of sale for the two settings are arbitrarily close to each other. However, the payoffs from these periods (which are in excess of $\underline{b}$) arrive at the beginning of the renegotiation equilibrium but at the end of the spot contracting equilibrium. With a discount rate of β which is bounded away from 1, this implies that the punishment payoffs in the spot contracting game are lower than those in the renegotiation game, and the seller in the renegotiation game would strictly prefer to deviate. □

Theorem 1 shows that, in a sense, the *endogenous* commitment power that a long lived seller has is not necessarily "increasing" in exogenous commitment.[3] "Exogenous" commitment power "increases" between the spot contracting and renegotiation settings, because the seller can write a contract including everything she could include in the spot contracting setting. However, shifting legal regimes in this way could make it *more* difficult to commit.

Like most repeated games, the potential punishments from the buyers are what makes commitment feasible. The key feature that drives this result is that in some cases, punishments are *worse* in the spot contracting setting than they are in the commitment with renegotiation setting. The worse punishments, combined with the fact that deviation payoffs are the same, mean that a seller need not be as patient to commit in the spot contracting setting as compared to the commitment with renegotiation setting.

Appendix

Proof of Proposition 1. Payoffs of $\mu\bar{b}$ arise from implementing the full commitment outcome, and any payoffs between v and $\mu\bar{b}$ can be attained by decreasing prices for the high type.

Consider the continuation equilibrium starting from period $kT+1$ (when $k\in\mathbb{N}$) which is worst for the seller. Buyers must be best responding during this period. Because it is the worst continuation equilibrium the seller must be best responding as well. Thus, actions must follow an equilibrium of the stage game.

As the discount factor approaches 1, $\bar{\mu}_i$ as defined by Hart and Tirole (1988) in their Remark 3 approaches

$$1-\left(\frac{\bar{b}-\underline{b}}{\bar{b}}\right)^{i}.$$

Take n such that $1-\left(\frac{\bar{b}-\underline{b}}{\bar{b}}\right)^{n}\leq\mu<1-\left(\frac{\bar{b}-\underline{b}}{\bar{b}}\right)^{n+1}$. With renegotiation, when $n<T$ the monopolist sells to only high types for the first n periods, then sells to everyone. At the beginning of period k, the mass of unidentified high types is

$$(1-\mu)\left[\left(\frac{\bar{b}}{\bar{b}-\underline{b}}\right)^{n+1-k}-1\right],$$

and the profits that the monopolist earns from that period are

$$\bar{b}-\left[(1-\mu)+(1-\mu)\left[\left(\frac{\bar{b}}{\bar{b}-\underline{b}}\right)^{n-k}-1\right]\right]\bar{b}$$

Adding these profits to the $T-n$ periods of $\underline{b}$ and divide by T to get the per-period average, one obtains

$$\frac{T-n}{T}\underline{b}+\frac{n}{T}\bar{b}-\frac{1}{T}(1-\mu)\bar{b}\sum_{t=1}^{n}\left(\frac{\bar{b}}{\bar{b}-\underline{b}}\right)^{t-1}.$$

Next, suppose that $n\geq T$. Again, the mass of high types that the seller has not yet identified as high types at the beginning of period k is

$$(1-\mu)\left[\left(\frac{\bar{b}}{\bar{b}-\underline{b}}\right)^{T-k+1}-1\right],$$

so profits are

$$\bar{b}-\frac{1}{T}(1-\mu)\bar{b}\sum_{t=1}^{T}\left(\frac{\bar{b}}{\bar{b}-\underline{b}}\right)^{t-1}.$$

For δ marginally lower than 1, payoffs are slightly lower than v, so v is also attainable in a repeated game equilibrium. In an equivalent model, Schmidt (1993) solves for the generically unique equilibrium of the spot contracting game with no discounting. Accounting for differences in notation, it is shown that the seller separates with the same sequence of posteriors, although this happens at the end of the interaction. For $\delta\approx 1$, the seller earns the same profits as in the renegotiation case.

Thus, in both settings, the payoffs from the worst equilibrium for the seller are equal to v. □

[3] These results are related to work by Baker et al. (1994), Schmidt and Schnitzer (1995), and Pearce and Stacchetti (1998) in which either making more actions contractable or making objective signals more precise can actually be harmful to the principal in a hidden-action setting. Similarly, Kovrijnykh (2013) finds a non-monotonic relationship between payoffs and commitment power when studying how stochastic contract enforcement affects debt contracts.

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