Biased Contest Judges

Zachary Breig* Greg Kubitz†

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Abstract

We consider the design of a repeated contest in the presence of a potentially biased judge. Beliefs about the judge's bias may discourage contestants from exerting effort. In a repeated contest, the identity of a previous winner influences these beliefs. The contest designer is able to commit to a strategy of selectively and stochastically overruling the judge's ranking of the contestants. Overruling the judge can increase contestants' total expected effort and may improve or worsen expected outcomes for a type of contestant the judge is likely biased against.

Keywords— Biased contest, Contest design, Feedback, Discouragement

*Lecturer, School of Economics, University of Queensland. Email: z.breig@uq.edu.au.

†Senior Lecturer, School of Economics and Finance, Queensland University of Technology. Email: gregory.kubitz@qut.edu.au.

1 Introduction

Contests are frequently used to choose which employees to promote, which researchers to give grants to, and which teachers to reward. The designers of these contests often rely on a third party to evaluate the contestants: shareholders allow managers to promote employees, expert reviewers advise governments about which researchers to fund, and universities use teaching evaluations to give awards. However, these judges can be biased against certain types of contestants, reducing their incentive to exert effort. Beliefs about the judge's bias evolve as contestants observe the results of previous contests. In this paper, we study how a contest designer should use the recommendations of a potentially biased judge to maximize effort provision in a dynamic contest.

We model a repeated contest in which the contest designer cannot evaluate contestants' submissions. Instead, a judge observes the effort choices of contestants and privately ranks contestants for the designer. Contestants have observable characteristics that do not directly affect the quality of their submission, but can lead to bias from the judge. Specifically, the judge's ranking is determined both by the contestants' efforts and their types. When contestants believe that this bias makes them unlikely to win, they put forth less effort.

The focus of our analysis is on how the designer should use the ranking provided by the judge. Because the ranking is informative about contestants' effort provision, awarding the prize on the basis of this ranking incentivizes contestants to put forth effort. However, the ranking is also informative about the judge's bias, and respecting the ranking could lead to more pessimistic contestants and less effort in future periods. This is the central trade-off that the designer faces: committing to overrule the judge leads to less pessimistic beliefs about the judge in future rounds of the contest, but does so at the cost of incentives in the current round.

We find that overruling the judge is never beneficial when there are no future rounds of the contest. This is true both in a single-period contest and in the last period of a dynamic contest. Overruling the judge at this point weakens incentives in that period, and with no continuation game there are no benefits to improving contestants' beliefs.

For some initial beliefs about the judge's bias, the designer can increase the total expected effort by publicly announcing that the ranking of the judge will sometimes be overturned. This is possible when there is a moderate likelihood that the judge is biased. When the judge is likely unbiased, there is no benefit of overturning the judge as contestants always exert effort. When the judge is very likely to be biased toward one type, others never put in effort.

Intervention by the contest designer can increase total expected effort via two different mechanisms. First, if one outcome of a contest leads to contestants exerting more effort in the future, the designer can selectively overturn the judge's rankings to increase the likelihood of this outcome. Second, overturning the judge's rankings jams the signal of the contest outcome and can prevent at least one type of contestant from becoming discouraged regardless of previous contest outcomes.

When the judge is more likely to be biased against one type of contestant, then without intervention from the designer, that type of contestant is more likely to lose a given contest and subsequently become discouraged. The first type of intervention, decreasing the likelihood that a particular contestant loses, is beneficial to the disadvantaged contestant in most cases. However, the second type of intervention will typically hurt this contestant. The most effective way to jam the negative signal of losing a contest is to increase the likelihood this loss came from the designer's intervention rather than the judge's bias. While the intervention encourages this type of contestant to exert effort after a loss, it increases the chance this contestant loses the current contest.

The paper adds to the literature of the impact of bias on contestants' effort. In this paper, a higher likelihood of bias unambiguously decreases effort. This is consistent with the idea that an unbiased contest leads to the most effort from contestants on average. In particular, if contestants are asymmetric then it is often optimal to favor the weaker contestant, leveling the competition and increasing overall effort (Baye et al., 1993; Che and Gale, 2003; Terwiesch and Xu, 2008). More recently it has been shown that bias can increase total effort even when contestants are ex-ante symmetric (Franke et al., 2013; Drugov and Ryvkin, 2017; Barbieri and Serena, 2018; Fu and Wu, 2020).[1] Given the contest success function in the current paper, the designer prefers to minimize the perceived bias of the judge. However, if bias increases expected effort the designer would still be able to impact the perception of bias by deciding when to overturn the judge.

Deciding how or whether to bias a contest toward a specific type of contestant is related to the policy of affirmative action. Chowdhury et al. (2019) gives a comprehensive overview of these policies and how results from the contest literature, some of which are mentioned above, inform the efficacy of the policies. This paper provides complimentary insights. The contest designer cannot choose bias and does not care about fairness of outcomes for their own sake. Instead, faced with a judge that is potentially biased, the designer wants to manage beliefs about the bias in order to increase contestant effort. We show that this goal and the goal of affirmative action are only

[1] In dynamic contests, the bias imposed on one round may impact the effort choice in the others (Ridlon and Shin, 2013; Barbieri and Serena, 2018). Including bias in a later period may reduce effort in that period but increase effort in the prior round.

sometimes aligned. In particular, the effort maximizing intervention can reduce the probability that a disadvantaged contestant wins.

Contests have been shown to be valuable when the judge or evaluator has different preferences from the designer but does not discriminate between types of decisions or agents. Frankel (2014) shows that ranking mechanisms are max-min optimal when the bias of the judge is unknown. Our approach is closer to that of Letina et al. (2020), which shows that a contest is optimal for judges who are known to be lenient, in that the designer's goal is to affect the actions of a third parties (the contestants) rather than the judge alone. However, rather than focusing on a bias which treats all contestants symmetrically, this paper focuses on a judge which favors certain *types* of agents.

Previous research has studied the impact of feedback on effort choice in dynamic contests. When the feedback gives information about relative position in the contest, a motivation effect may increase output prior to feedback and a discouragement effect may lower effort after (Aoyagi, 2010; Ederer, 2010). Private feedback about relative position can dominate public information when uncertainty is high (Mihm and Schlapp, 2019). When feedback yields information about contestant abilities, sandbagging can lead to less effort from higher types prior to feedback, lowering expected effort (Zhang and Wang, 2009; Kubitz, 2020). In this paper, contestants are short-lived and therefore are only motivated by current beliefs, not by how current actions may affect future beliefs. This allows us to study how the release of interim information affects subsequent behavior without introducing signalling concerns into contestants' strategies. Our novel type of intervention by the designer seeks to reduce the feedback which discourages contestants.

More generally, the paper is related to the literature of information design where the contest designer con commit to a information disclosure rule over potential asymmetries between the contestants.[2] A majority of work focuses on optimal information disclosure about contestants' abilities or the number of contestants prior to a single contest (Feng and Lu, 2016; Zhang and Zhou, 2016; Zheng et al., 2018, 2019; Chen, 2020). Lu et al. (2018) and Serena (2019) identify optimal disclosure when only full or no disclosure is available. In our setting the designer only has the coarse tool of not following the recommendations of the judge to impact the beliefs of contestants competing in the second round. Using that tool has an endogenous cost of weakening incentives in the first contest.

The paper proceeds as follows. Section 2 describes the economic setting. Section 3 characterizes the solutions both the static and dynamic versions of the model. Section 4 discusses assumptions of

[2]See Bergemann and Morris (2019) and Kamenica (2019) for recent overviews of the information design literature.

the model and concludes.

2 Model

A designer holds a contest for two short lived contestants each period for T periods. In each period, one contestant of type A and one of type B participates. The type of each contestant is publicly observable. During the period t in which they participate, each contestant can choose $a_{i,t} \in \{0,1\}$ for that period. If they choose $a_{i,t} = 1$ (which we refer to as "putting forth effort"), they pay a cost c, and otherwise pay no cost. We present the optimal contest design for $T = 1$ and $T = 2$. The (predetermined) prize in each period is v.

Effort levels are observed by a judge who ranks the contestants and sends the ranking privately to the designer. The same judge evaluates participants in each contest. This judge is not strategic (or at least has no preferences over outcomes of the contest) but may evaluate the contestants' output with bias.[3] We say that the judge either *favors type A*, *favors type B*, or is *unbiased*. These characteristics are mutually exclusive and both the designer and all contestants have a common prior for these events of $p_{A,1}$, $p_{B,1}$, and $1 - p_{A,1} - p_{B,1}$ respectively.

The likelihood that the judge ranks a particular contestant first in any period depends on both contestants' efforts and the (potential) bias of the judge. In particular, if both contestants put forth effort then a judge biased toward type i ranks that type higher with probability one, and an unbiased judge reports each ranking with equal probability. If only one contestant puts forth effort the judge ranks this contestant first with probability one. Finally, if neither type puts forth effort, the judge reports each ranking with equal probability.[4]

The designer has a discount factor of $\delta < 1$ and maximizes the discounted sum of effort provided by the contestants. Prior to the start of the first round, she can set the likelihood that each contestant

[3]The assumption that the judge doesn't act strategically seems strong, but there are many cases in which it is realistic. For instance, if submissions are judged by a computer program or algorithm which produces rankings, the designer need not worry about strategic interference by the judge. Alternatively, the designer might also act as a judge knowing that they might display subconscious bias, but not know what or how strong that bias is.

[4]A more general model would have the judge observing the following score in each period for an agent of type i:

$$b_i a_{i,t} + \varepsilon_{i,t},$$

where b_i is an unchanging bias parameter, $\varepsilon_{i,t}$ is a noisy shock, and the judge reports the contestant with the highest score to the designer. The reported model is then a limiting case of this structure, where $b_A > b_B$ with probability $p_{A,1}$, $b_B > b_A$ with probability $p_{B,1}$, and the variance of $\varepsilon_{i,t}$ goes to 0.

receives the prize conditional on the judge's report. Let $\gamma_{i,t}$ be the probability that the contestant of type i wins the prize in period t given the judge ranking them first. This choice is public and the designer has the power of full commitment.[5]

In what follows, we focus on the Perfect Bayesian Equilibria (PBE) of the game between the designer and all contestants. A strategy of the designer is $\Gamma = (\gamma_{A,1}, \gamma_{B,1}, \gamma_{A,2}, \gamma_{B,2})$. Strategies of each contestant consist of an action choice in the round each is active for each history, $a_{i,t}(h_t)$. Histories in each period are: $h_1 = \Gamma$ and $h_2 \in \{(\Gamma, A), (\Gamma, B)\}$ where the second argument indicates which type won the first contest. Beliefs about the judge's bias for each history are denoted $p_{i,t}(h_t)$. Note that $p_{i,1}(\Gamma) = p_{i,1}$ for all Γ.

For a given prior belief about the judge's bias, $(p_{A,1}, p_{B,1})$, a PBE consists of strategies for the designer and each contestant such that

1. Given the strategy of the designer, Γ, the contest outcomes, and conjectures about the actions of contestants, $\hat{a}_{i,t}$, beliefs about the judge's bias are updated using Bayes rule where possible.[6]

2. Actions, $a_{i,t}$, maximize the payoff of each contestant, $i = A, B$, in each period, $t = 1, 2$, given history, h_t, beliefs about the judges bias, $p_{i,t}(h_t)$, and conjectures of the action of the opposing contestant $\hat{a}_{-i,t}$.

3. The designer's strategy, Γ, maximizes the discounted expected effort of the contestants, $\mathbb{E}[a_{A,1} + a_{B,1} + \delta(a_{A,2} + a_{B,2})]$, given 1 and 2.

4. Actions equal conjectures, $a_{i,t} = \hat{a}_{i,t}$.

Without loss of generality, we will assume that $c = 1$, since all contestants' incentives are only related to the ratio between v and c rather than the levels. In cases in which one or more of the contestants is indifferent about whether to put forth effort, we assume that the contestant breaks that indifference in favor of putting forth effort.[7] Finally, we will assume that $v > 2$ to focus on situations in which it's possible for both contestants to put forth effort in equilibrium. Due to the symmetric nature of the problem, our analysis in Section 3 is done under the assumption that the judge is more

[5]We discuss the implications of commitment and what equilibria survive without it in Section 4.

[6]The only players who could cause a probability zero event to happen are the contestants in period 1. Since their payoffs do not depend on beliefs in the future, they will never have an incentive to change actions. Therefore it never matters what beliefs are after the zero probability event.

[7]This assumption is made primarily for convenience. It eliminates cases of multiple equilibria over the measure-zero set of priors for which the designer's optimal choices must involve indifferent contestants.

likely to be biased toward contestants of type A, i.e. $p_{A,1} \geq p_{B,1}$. Results for when $p_{B,1} > p_{A,1}$ directly follow.

3 Results

3.1 One Period Solution

Before presenting the solution to the one period problem, we define the *discouragement* due to bias as

$$D(v, p_{A,1}, p_{B,1}) = \frac{2}{v} \cdot \frac{1}{1 + p_{B,1} - p_{A,1}}.$$

Discouragement plays an important role in the solutions of both the one period and two period models. Simply put, $D(v, p_{A,1}, p_{B,1})$ captures the discouraging effect that the judge's potential bias has on the less favored type. Discouragement increases with the difference in the likelihood that the two types are favored. However, this discouraging effect is weaker when the prize v is higher: even if a contestant thinks they are unlikely to win, they will still put forth effort if the prize is high enough.

With this definition of discouragement, describing the equilibrium of the model in which $T = 1$ is straightforward.

Theorem 1 *In any equilibrium of the single period model, both types of contestant put forth effort if*

$$D(v, p_{A,1}, p_{B,1}) \leq 1.$$

Otherwise, only type A puts forth effort. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort.

A graphical representation of the equilibrium described in Theorem 1 can be found in Figure 1. When one contestant is very likely to be favored while the other is very unlikely to be favored, only the contestant who is more likely to be favored puts forth effort. Essentially, the disparate beliefs have a discouraging effect on the contestant with a lower prior; he knows that even if he puts forth effort, he's unlikely to win. Discouragement happens for a wider range of initial beliefs when the prize is smaller. On the other hand, when $p_{A,1}$ and $p_{B,1}$ are similar and the other contestant is putting forth effort, switching from $a_{i,1} = 0$ to $a_{i,1} = 1$ increases the perceived likelihood of winning by roughly

50%, so both contestants put forth effort.

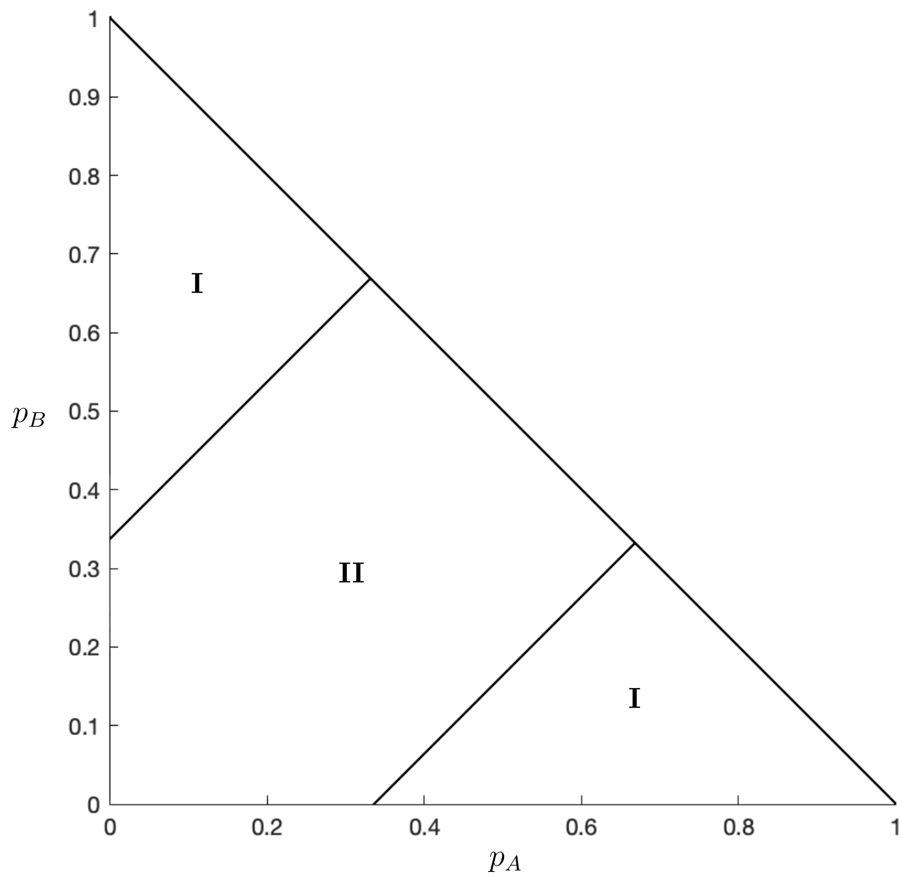


Figure 1: Solution to the single period model when $v = 3$. Both types of contestant put forth effort in the region labeled II, while only one type puts forth effort in the region labeled I.

One notable feature of the equilibrium described in Theorem 1 is that the designer has no use for setting $\gamma_{A,1}$ or $\gamma_{B,1}$ less than one. While the designer may be able to manipulate contestants' beliefs for the future by not giving the prize to the contestant the judge chose, doing so comes at a cost of weaker incentives in the current period. Because there is no continuation game when $T = 1$, lowering $\gamma_{A,1}$ and $\gamma_{B,1}$ can only hurt the designer and reduce incentives to put forth effort.

3.2 Two Period Solution

In the static model, the designer had no reason to give the prize to the contestant which was not reported as the winner by the judge. Essentially, giving the prize to someone other than the reported winner simply weakens incentives without any additional benefits. Making the model dynamic introduces a new feature which may make it beneficial to give the prize to a different agent: reallocating the prize away from the reported winner can affect the beliefs of contestants in a later period, potentially causing them to put forth effort when they otherwise would not. Additionally, if one outcome from the first contest leads to higher output in the second contest, the designer can increase the probability of this outcome.

Figure 2 shows how beliefs update between the first and second period for the case of $\gamma_{1,A} = \gamma_{1,B} = 1$ when a contestant of type B wins in the first period. The starting point of the arrow indicates the prior beliefs, while the end of the arrow shows where beliefs are at the beginning of the second period. These posterior beliefs are on the vertical axis because there is zero possibility that the judge favors type A: given that both types put forth effort in the first period, observing type B win means that the judge either favors type B or is unbiased.

In the case of the blue arrow in Figure 2, beliefs updating does not cause a problem; even after the contestant of type B wins in the second period, both second period contestants think it is relatively likely that the judge is unbiased and are still willing to put forth effort. However, for both red arrows this updating process leads to pessimistic beliefs about contestants of Type A. In these cases, contestants of Type A would be unwilling to put forth effort in the second period.

More generally, from Bayes' rule, updated beliefs when both contestants are conjectured to exert effort in the first round must satisfy

$$\begin{aligned} p_{A,2}(\Gamma, A) &= \frac{\gamma_{A,1} p_{A,1}}{\frac{1}{2}\gamma_{A,1}(1 + p_{A,1} - p_{B,1}) + \frac{1}{2}(1 - \gamma_{B,1})(1 + p_{B,1} - p_{A,1})} \\ p_{B,2}(\Gamma, A) &= \frac{(1 - \gamma_{B,1}) p_{B,1}}{\frac{1}{2}\gamma_{A,1}(1 + p_{A,1} - p_{B,1}) + \frac{1}{2}(1 - \gamma_{B,1})(1 + p_{B,1} - p_{A,1})}. \end{aligned} \tag{1}$$

Setting $\gamma_{A,1}$ or $\gamma_{B,1}$ to less than one introduces noise into the learning process and causes contestants to update less. Specifically, the posterior belief about bias toward the losing type of contestant is not as low and toward the winning type of contestant is not as high. For $\gamma_{A,1} = \gamma_{B,1} = \frac{1}{2}$, the prize is assigned in a way which is uncorrelated with the judge's report and beliefs do not update at all.

Lemma 1 *The following relationships between posterior beliefs about the judges bias and the designer's strategy hold when both contestants are conjectured to exert effort in the first contest and $\gamma_{i,1} > 1/2$ for $i = A, B$.*

$$\frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{-i,1}} > \frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{i,1}} > 0 \text{ and } \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{i,1}} < \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{-i,1}} < 0.$$

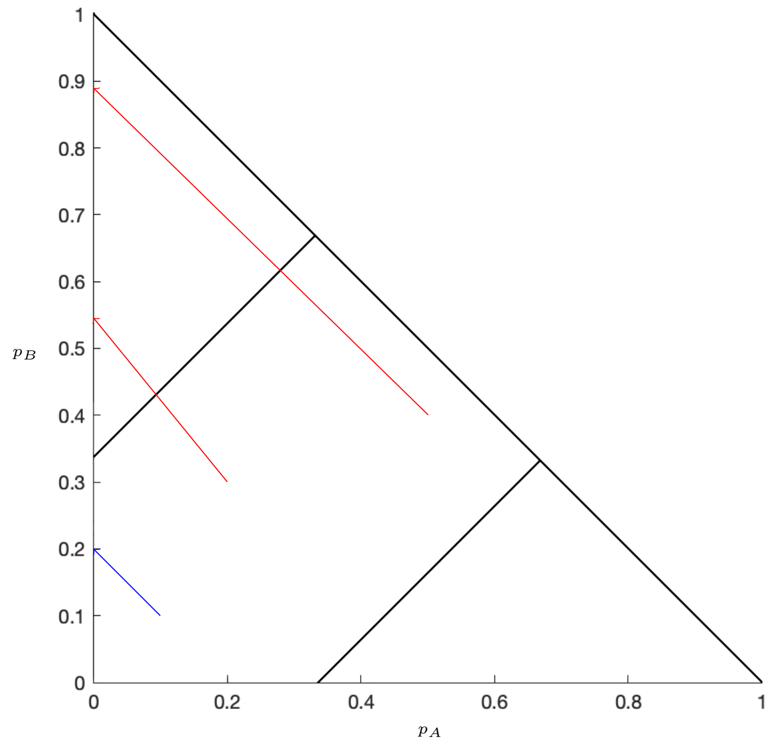


Figure 2: Updated beliefs after the type B contestant wins in the first period without intervention from the designer. For the blue arrow, both types of contestant put forth effort in the second period. For the red arrows, only contestants of type B put forth effort in the second period.

Given that the contestant of type B is putting forth effort in period t, a contestant of type A only puts forth effort if

$$\gamma_{A,t} + \gamma_{B,t} - 1 \geq \frac{2}{v} \cdot \frac{1}{1 + p_{A,t}(h_t) - p_{B,t}(h_t)}. \tag{2}$$

This captures the features which are payoff-relevant to a contestant in a given period. For a contestant of type i, higher values of $p_{i,t}(h_t)$, $\gamma_{i,t}$, and $\gamma_{j,t}$ make it more valuable to put forth effort, while $p_{i,t}(h_t)$ makes effort less valuable.

The cost of lowering $\gamma_{A,t}$ or $\gamma_{B,t}$ is weaker incentives in period t, while the benefit arises from "better" beliefs (from the point of view of the designer) in later periods. Thus, in the final period the designer does not benefit from lowering either value, and sets both equal to one.

Lemma 2 *It is optimal for the designer to set $\gamma_{A,2} = \gamma_{B,2} = 1$ in the second period.*

Beliefs about whom the judge favors are updated after observing whom the prize is given to when both types are putting forth effort in the first period. However, this is not true when at least one type doesn't put forth effort. In this case, beliefs do not update for either type of contestant no matter whom the prize is given to. Given that only one contestant put forth effort, the judge reports that they are the winner with probability one, and the only way that the other contestant could be awarded the prize is if the designer went against the judge's recommendation. Combining this with the fact that $\delta < 1$, it is never optimal for the designer to decrease $\gamma_{A,1}$ or $\gamma_{B,1}$ to increase effort in future periods at the cost of lowering effort today. Thus, for all equilibria where prior beliefs are in region II, the designer's strategy must be such that both contestants exert effort in the first period. We can combine the inequalities given in (2) in period one for both types to get this condition which characterizes the minimum levels of $\gamma_{A,1}$ and $\gamma_{B,1}$ which ensure that both constants put forth effort in the first period.

$$D(v, p_{A,1}, p_{B,1}) \leq \gamma_{A,1} + \gamma_{B,1} - 1 \tag{IC-1}$$

While beliefs about whom is favored are updated when the first period's prize is awarded, whether those posteriors still allow for effort provision in the second period depends on both initial beliefs and the designer's strategy, Γ. We combine the updating rules (1) with the incentive constraints given in inequality (2) to find

$$\gamma_{A,1} - \frac{\left(1 - \frac{2}{v}\right)(1 + p_{B,1} - p_{A,1}) - 2p_{B,1}}{\left(1 - \frac{2}{v}\right)(1 + p_{A,1} - p_{B,1}) + 2p_{A,1}} \gamma_{B,1} \leq 1 \tag{BC-A}$$

$$\gamma_{B,1} - \frac{\left(1 - \frac{2}{v}\right)(1 + p_{A,1} - p_{B,1}) - 2p_{A,1}}{\left(1 - \frac{2}{v}\right)(1 + p_{B,1} - p_{A,1}) + 2p_{B,1}} \gamma_{A,1} \leq 1. \tag{BC-B}$$

These *belief constraints*, (BC-A) and (BC-B), characterize the values of $\gamma_{A,1}$ and $\gamma_{B,1}$ for which A and B (respectively) put forth effort in the second period conditional on losing. Specifically, they ensure that the beliefs about type i do not fall too much upon observing type i lose. In principle we must also check that beliefs about type i do not fall too much upon observing type i win. However, one can show that beliefs only fall in this way if $\gamma_{A,1} + \gamma_{B,1} < 1$. Such values of $\gamma_{A,1}$ and $\gamma_{B,1}$ would violate (IC-1), and thus will never hold in equilibrium.

We interpret the coefficient on $\gamma_{B,1}$ in (BC-A) and on $\gamma_{A,1}$ in (BC-B) as the *confidence* that types A and B respectively have that the judge does not favor the other type. It will be convenient to denote this as

$$C_A(v,p_{A,1},p_{B,1}) = \frac{\left(1-\frac{2}{v}\right)(1+p_{B,1}-p_{A,1})-2p_{B,1}}{\left(1-\frac{2}{v}\right)(1+p_{A,1}-p_{B,1})+2p_{A,1}}$$

and

$$C_B(v,p_{A,1},p_{B,1}) = \frac{\left(1-\frac{2}{v}\right)(1+p_{A,1}-p_{B,1})-2p_{A,1}}{\left(1-\frac{2}{v}\right)(1+p_{B,1}-p_{A,1})+2p_{B,1}}.$$

We say that type i is *confident* if $C_i(v,p_{A,1},p_{B,1}) \geq 0$. This cutoff is relevant because when it is satisfied, a contestant of type i is willing to put forth effort in the second round after observing their type lose *even when* $\gamma_{A,1}=\gamma_{B,1}=1$.[8] Furthermore, for more negative values of $C_A(v,p_{A,1},p_{B,1})$ and $C_B(v,p_{A,1},p_{B,1})$, the set of values of $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy (BC-A) and (BC-B) is smaller.

The incentive constraint for the first period (IC-1) and belief constraints for the second period (BC-A) and (BC-B) capture the designer's problem. Lowering $\gamma_{A,1}$ or $\gamma_{B,1}$ may help to satisfy (BC-A) or (BC-B). However, doing so cannot come at the cost of violating (IC-1). Thus, the solution to the designer's problem will depend on the priors, which determine whether (BC-A) and/or (BC-B) can be satisfied without violating (IC-1).

Our first result characterizes the equilibrium of the two period game when neither type A nor B is confident. In this case, without intervention from the designer, the contestant whose type lost in the first period would never put forth effort in the second period. We suppress the arguments of the discouragement and confidence functions for the remainder of the section for expositional ease.

Theorem 2 *Suppose that no type of contestant is confident.*

- *If $D \leq \frac{(1+C_B)(1+C_A)}{1-C_AC_B}$, then in any PBE both types of contestant put forth effort in both periods and the designer occasionally overturns the judge.*
- *If $\frac{(1+C_B)(1+C_A)}{1-C_AC_B} < D \leq 1+C_B$, then in any PBE only contestants of type B put forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{A,1}=1$, and $\gamma_{B,1}$ to the minimum level possible so that both types put forth effort in the first period.*
- *If $1+C_B < D \leq 1+C_A$, then in any PBE only contestants of type A put forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{B,1}=1$, and $\gamma_{A,1}$ to the minimum level possible so that both types put forth effort in the first period.*

[8] This can be seen by considering (BC-A). When $C_A(v,p_{A,1},p_{B,1}) \geq 0$, the inequality holds for *any* values of $\gamma_{A,1}$ and $\gamma_{B,1}$ that are between 0 and 1.

- *If $1 + C_A < D \leq 1$, then in any PBE neither type of contestant puts forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*

- *If $1 < D$, then in any PBE contestants of type A put forth effort in both periods but contestants of type B do not put forth effort in either period.*

The way in which equilibria vary with primitives in Theorem 2 is intuitive. When discouragement is low and contestants are confident, the designer is able to induce both types of contestant to put forth effort in both periods by occasionally overturning the judge. For higher levels of discouragement or lower levels of confidence, the designer is able to extract correspondingly less effort from the contestants.

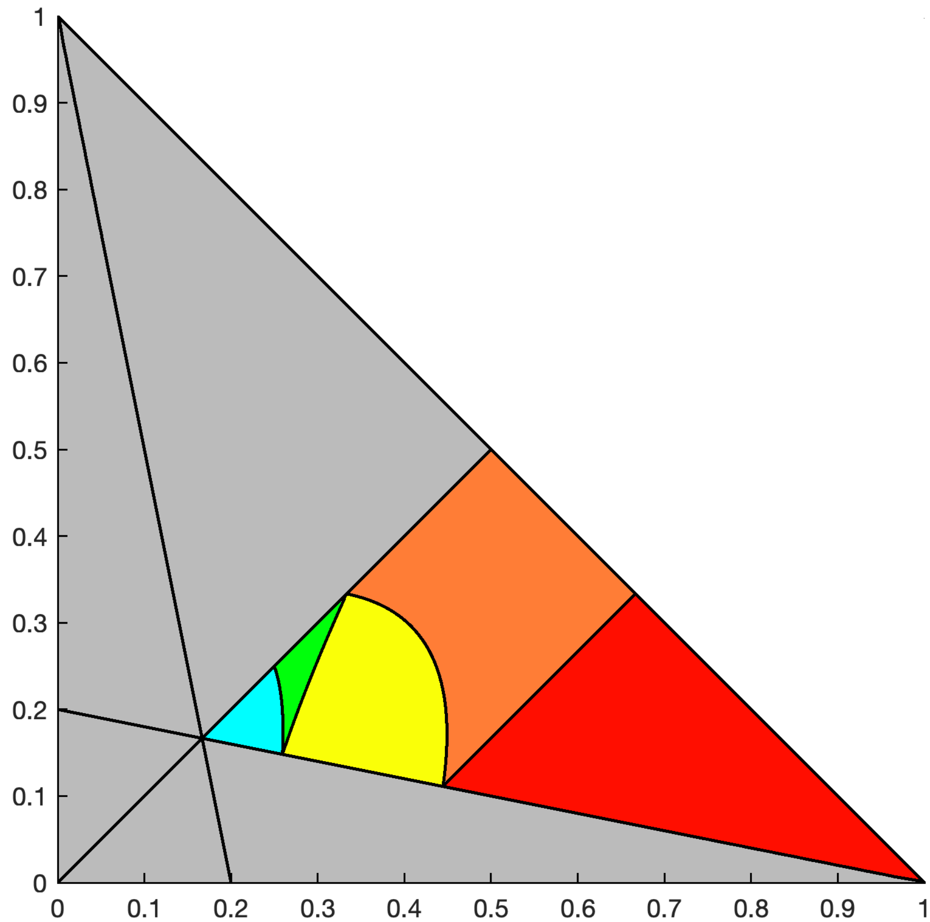
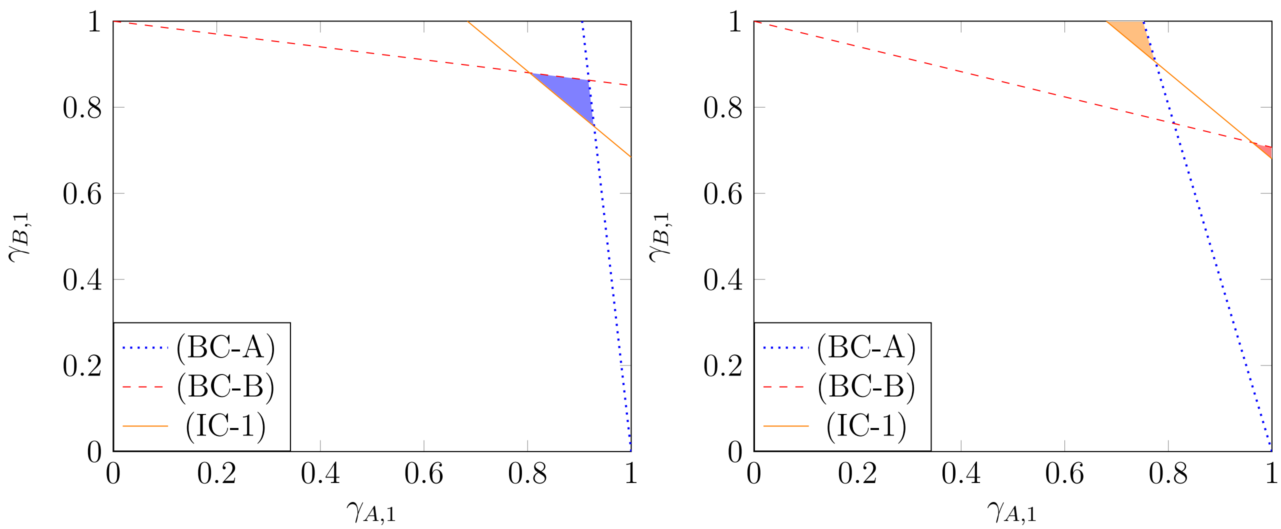


Figure 3: The colorful regions of this figure show how the form of the equilibrium varies with priors when $v = 3$ and neither type is confident. Total effort provision decreases from left to right. The designer overturns the judge in the light blue, green, and yellow regions.



(a) $p_{A,1} = 0.225$, $p_{B,1} = 0.2$, $v = 3$

(b) $p_{A,1} = 0.3$, $p_{B,1} = 0.28$, $v = 3$

Figure 4: This figure shows the interaction of three constraints for two sets of parameters. In panel (a), the blue triangle represents the values of $(\gamma_{A,1}, \gamma_{B,1})$ which satisfy (BC-A), (BC-B), and (IC-1). In panel (b), values within the orange triangle satisfy (BC-A) and (IC-1), while values in the red triangle satisfy (BC-B) and (IC-1). No values in panel (b) satisfy all three constraints.

The sets of parameters for associated with the types of equilibria described in Theorem 2 can be found in Figure 3. The non-grey region corresponds to parameter values such that neither type is confident and $p_{A,1} \geq p_{B,1}$. Each color represents a different form of the equilibria from Theorem 2: $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$ is in light blue, $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B$ is in green, $1 + C_B < D \leq 1 + C_A$ is in yellow, $1 + C_A < D \leq 1$ is in orange, and $1 < D$ is in red. Note that when $C_A < 0$, then $p_A \geq p_B$ implies that $C_A \geq C_B$, so Theorem 2 characterizes all of these cases.

Figure 4 graphically shows how the three constraints determine what values of $\gamma_{A,1}$ and $\gamma_{B,1}$ should be chosen for fixed values of $p_{A,1}$, $p_{B,1}$, and v. The graph in panel (a) corresponds to the case in which neither type of contestant is confident but $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$. The shaded blue triangle is the set of γ which satisfies (IC-1), (BC-A), and (BC-B). Thus, *any* values of $\gamma_{A,1}$ and $\gamma_{B,1}$ which lie within this triangle guarantee that both types of contestant put forth effort in both periods.

One portion of Theorem 2 which merits further discussion is the range of priors for which $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B$. The fact that $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D$ indicates that the designer cannot simultaneously incentivize *both* types to put forth effort conditional on losing, but since $D \leq 1 + C_B$ and $D \leq 1 + C_A$, the designer can incentivize *either*. This can be seen in panel (b) of Figure 4, where values of $\gamma_{A,1}$ and $\gamma_{B,1}$ in the orange region guarantee that the contestant of type A puts forth effort conditional on losing in the first period, while this is true for type B in the red region. Since only one type will put forth effort conditional on losing, the designer prefers to choose the type which can be induced to lose the most often, because this increases the likelihood both types exert effort in the second period. To do this, the designer's intervention ensures that the type which is disfavored *a priori* puts in effort in the second period even after that type loses the first period. Furthermore, the optimal way to do this is to minimize the chance that this type wins conditional on both parties still having an incentive to put forth effort in the first period (in this case, setting $\gamma_{B,1}$ to the minimum value which does not violate (IC-1)). The optimal choice of $\gamma_{A,1}$ and $\gamma_{B,1}$ in this case is exactly the point where $\gamma_{A,1} = 1$ on the line representing (IC-1).

Reducing $\gamma_{B,1}$ while keeping $\gamma_{A,1} = 1$ has two impacts that benefit the designer. The first is the direct impact of having contestant B lose more often. As type A will not put in effort in the second period after a loss, the designer wishes to minimize the probability that this happens. Secondly, the best way to make type B more optimistic about the second period after losing the first period is to reduce $\gamma_{B,1}$. This can be seen from Lemma 1. Decreasing $\gamma_{B,1}$ both increases $p_{B,2}(\Gamma, A)$ and decreases $p_{A,2}(\Gamma, A)$ at a faster rate than decreasing $\gamma_{A,1}$. Intuitively, a bad outcome for B can more easily attributed to (bad) luck for lower values of $\gamma_{B,1}$; a loss by type B is more likely to have been caused by the judge being overruled by the designer when B was actually ranked higher.

The next result characterizes the equilibrium of the two period game when type A is confident but type B is not.

Theorem 3 *Suppose that contestants of type A are confident but contestants of type B are not confident.*

- *If $D \leq 1 + C_B$, then in any PBE both types of contestant put forth effort in both periods and the designer occasionally overturns the judge.*
- *If $1 + C_B < D \leq 1$, then in any PBE both types of contestant put forth effort in the first period, but only contestants of type A put forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{B,1} = 1$, and $\gamma_{A,1}$ to the minimum level possible so that both types put forth effort in the first period.*
- *If $1 < D$, then in any PBE contestants of type A put forth effort in both periods but contestants of type B do not put forth effort in either period.*

Given that type A is confident, the designer does not need to intervene in order for contestants of type A put forth effort in both periods. Instead, the form of the equilibrium depends on the discouragement and confidence of type B. When confidence is not too low, only a small decrease in $\gamma_{B,1}$ is necessary for (BC-B) to be satisfied. This small decrease does not lead to a violation of (IC-1) when D is not too high. For lower confidence levels, the intervention necessary to incentivize second period effort from type B conditional on them losing in the first period becomes infeasible. Finally, if discouragement is too high, type B is not even willing to put forth effort in the first period.

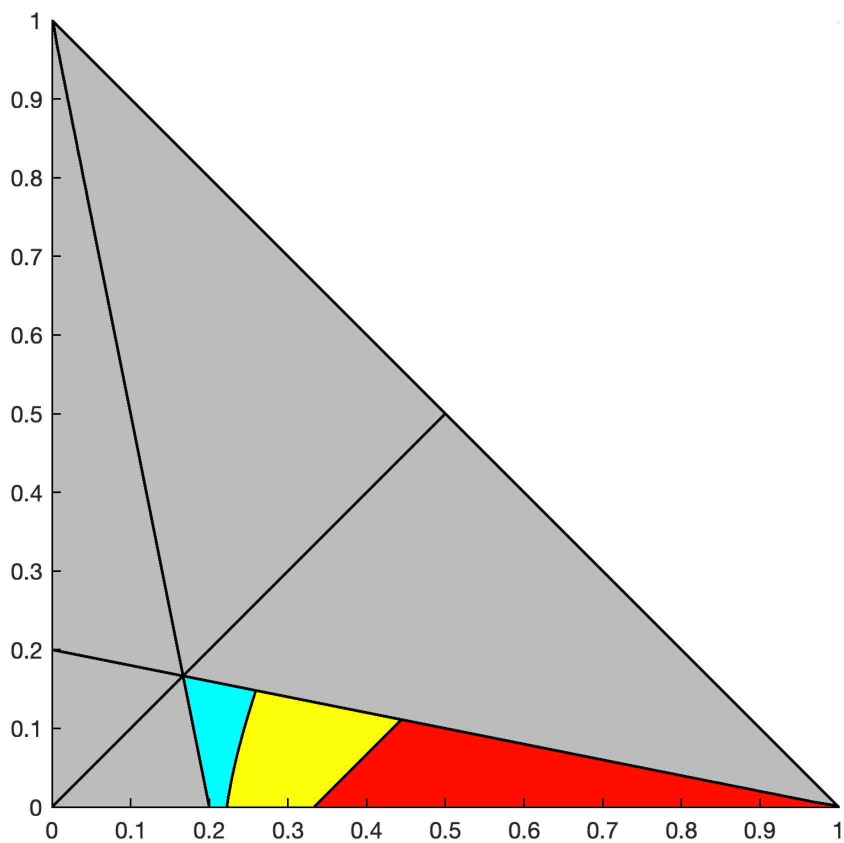


Figure 6: The colorful regions of this figure show how the form of the equilibrium varies with priors when $v = 3$, type A is confident, and type B is not. Total effort provision decreases from left to right. The designer overturns the judge in the light blue region.

A graphical depiction of the of the regions described in Theorem 3 can be found in Figure 6. The non-grey region corresponds to parameter values such that type A is confident, type B is not, and $p_{A,1} \geq p_{B,1}$. Each color represents a different form of the equilibria from Theorem 2: $D \leq 1 + C_B$ is in light blue, $1 + C_B < D \leq 1$ is in yellow, and $1 < D$ is in red.

Finally, we characterize the equilibrium of the two period game when *both* types are confident.

Theorem 4 *Suppose that both types of contestant are confident. Then both types put forth effort in both periods. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*

This result is straightforward. Since both types are confident, both types have the incentive to put forth effort in the second period conditional on their type losing in the first period. This is true even without interference from the designer. The only point left to check is that both types have the incentive to put forth effort in the first period, but (IC-1) is always satisfied with $\gamma_{A,1} = \gamma_{B,1} = 1$ when both types are confident. The region for which both types of contestant are confident and $p_{A,1} \geq p_{B,1}$ can be seen in Figure 7.

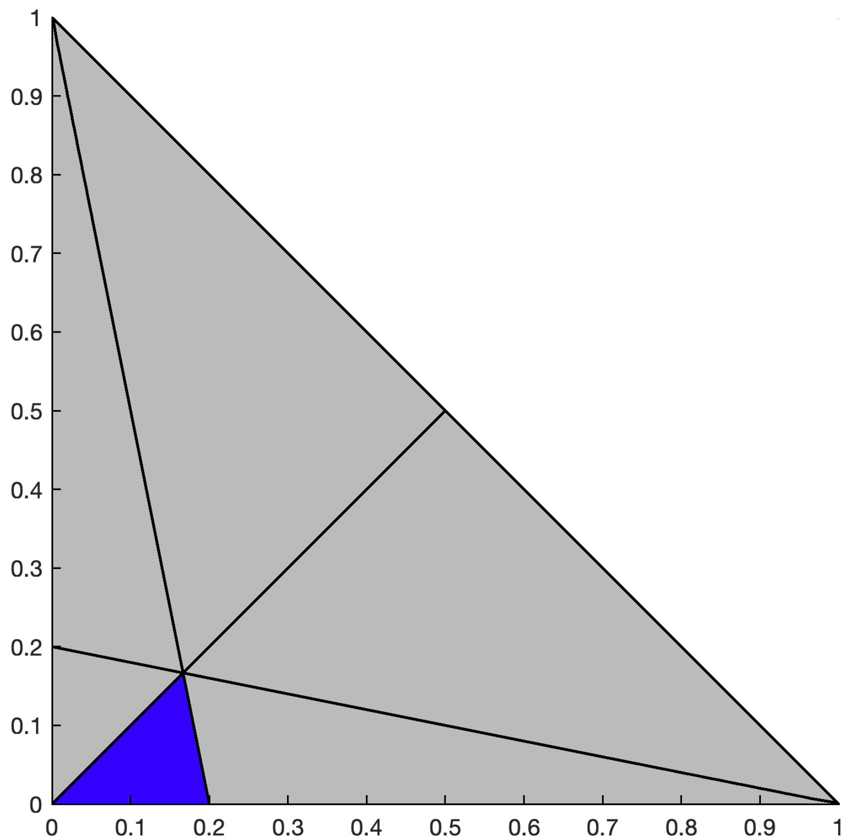


Figure 7: The blue region of this figure shows the priors for which both types are confident when $v = 3$.

4 Discussion

We study a dynamic contest in which sequentially arriving contestants learn about the contest judge's bias by observing who has won in the past. We allow a contest designer to manipulate who is awarded a prize in order to manage contestants' beliefs and maximize effort. We show that for some prior beliefs about the judge's bias, the designer can increase total expected effort. Specifically, there are beliefs for which this intervention can ensure that all contestants put forth effort conditional on losing when neither (light blue area in Figure 3) or only one (light blue area in Figure 6) would without it. For priors where intervention can only induce one type of contestant to exert effort in the contest, she prefers to choose the one which is more likely to face bias from the judge (green area in Figure 3). The optimal intervention in this case lowers this type of contestants likelihood of receiving the prize in the first round even further. Lastly, where no intervention can prevent the type more likely to face bias from not exerting effort after losing the first contest, we show the designer can benefit by reducing the likelihood this type loses the first round (yellow area in Figures 3 and 6).

The results from Theorem 2 demonstrate the difficulty caused by the signal space being restricted to the report of the winner. Regardless of the intervention chosen in our setting, there are at most two signals that contestants can observe after the first round. Manipulations of the judges report impact the informativeness of each signal and how often each occurs. Interventions to maximize effort can harm the type of contestant more likely facing bias. In this case, the designer wishes to reduce the informativeness of the signal that is more likely to happen, namely that this type loses. This is done by artificially increasing the likelihood of this event - overturning rankings when this disadvantaged type is chosen by the judge.

In settings where a more continuous measure of quality could be sent (e.g. one in which a continuous "score" is generated and observed from each contestant's submission) this incentive on the part of the designer need not be present. More generally, the question of what signals should be revealed in various environments is likely to be a fruitful path for future research which draws upon both the dynamic contest and information design literatures. We show how the designer is able to manipulate information in this realistic setting where only basic outcomes of past contests are publicly available.

The model has been presented as an analysis of contest design with a biased judge, but it could also be interpreted as a model in which contestant types have uncertain ability (for example, each type of contestant could represent a different technological or strategic approach). When making their effort choices, contestants observe how successful previous contestants of their type were. Optimal interventions in this model would be the same if the designer is indifferent between effort provision from contestants with different ability levels. If effort from high ability contestants was preferred then interventions that always induce the high ability contestant to exert effort in the second contest may be preferred over one where instead the low ability contestant would always exert effort in the second contest.

We conclude by discussing a few of the assumptions of the model.

The contest designer has the power to publicly commit at the beginning of the game to a strategy with randomization that depends on the judge's report. This ability to commit is important at several points. If the randomization is chosen *after* contestants choose their effort in the first period, the designer would sometimes benefit from ignoring (IC-1) and assigning the prize in a way unrelated to the judge's report. Furthermore, without the commitment to respect the outcome of the randomization, equilibria with intervention would only remain when total effort is the same in the second contest given either outcome from the first contest (light blue area of Figures 3 and Figure 6).

We assume that the judge's bias only affects the outcome when both contestants put forth effort, making the bias a *complement* to effort. An alternative assumption would be for bias to be a *substitute* for effort, such that a judge that favors type i reports type i as the winner with probability one when either both players put forth effort or neither player puts forth effort. While equilibria are different and may involve mixed strategies in this model, we expect that many of the insights of our results will still be applicable: The designer still trades off between stronger incentives in the present and better beliefs in the future,

Our analysis also assumes the designer knows the precise common prior beliefs of the contestants. The assumption is easy to (partially) relax without qualitatively changing the results. Some values of $\gamma_{A,1}$ and $\gamma_{B,1}$ solve the designer's problem for a range of the contestants' prior beliefs. Thus, as long as the designer's beliefs about the contestants' priors fall within this range, the same choice of of $\gamma_{A,1}$ and $\gamma_{B,1}$ solve the designer's problem. Inducing additional uncertainty on the part of the designer is an interesting pathway for future research.

The restriction to a two period model is another assumption which should be relaxed in future work. This restriction brings with it meaningful consequences: the designer's problem is centered around mitigating the effects of a loss on one type of contestant. This leaves no space for intuitive strategies such as occasionally awarding the prize to the type which has lost several past contests in order to make them less pessimistic. We expect such strategies to be valuable when the model is extended to more periods.

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A Proofs

Theorem 1 *In any equilibrium of the single period model, both types of contestant put forth effort if*

$$D(v, p_{A,1}, p_{B,1}) \leq 1.$$

Otherwise, only the type more likely to be favored puts forth effort. The designer sets $\gamma_{A,1}$ *and* $\gamma_{B,1}$ *high enough so that neither contestant has the incentive to stop putting forth effort.*

Proof. If the contestant of type B is not putting forth effort, the type A contestant puts forth effort if

$$\mathbb{P}(A \text{ receives prize} | a_{A,1} = 1, a_{B,1} = 0)v - 1 \geq \mathbb{P}(A \text{ receives prize} | a_{A,1} = 0, a_{B,1} = 0)v.$$

We can substitute in the judge's decision rule to get

$$\gamma_{A,1} v - 1 \geq \left[\frac{1}{2}\gamma_{A,1} + \frac{1}{2}(1 - \gamma_{B,1})\right] v.$$

which simplifies to

$$\frac{2}{v} \leq \gamma_{A,1} + \gamma_{B,1} - 1.$$

Notice that this equality also holds for type B putting forth effort when A is not.

If the contestant of type B is putting forth effort, the type A contestant puts forth effort if

$$\mathbb{P}(A \text{ receives prize} | a_{A,1} = 1, a_{B,1} = 1)v - 1 \geq \mathbb{P}(A \text{ receives prize} | a_{A,1} = 0, a_{B,1} = 1)v.$$

We can substitute in the judge's decision rule to get

$$\left[p_{A,1}\gamma_{A,1} + p_{B,1}(1 - \gamma_{B,1}) + \frac{1}{2}(1 - p_{A,1} - p_{B,1})(1 + \gamma_{A,1} - \gamma_{B,1})\right] v - 1 \geq (1 - \gamma_{B,1})v$$

which simplifies to

$$\frac{2}{v}\left(\frac{1}{1 + p_{A,1} - p_{B,1}}\right) \leq \gamma_{A,1} + \gamma_{B,1} - 1$$

with a symmetric inequality holding for the contestant of type B.

Thus, increasing $\gamma_{A,1}$ and $\gamma_{B,1}$ increases the incentive to put forth effort for both contestants in all situations, so without loss of generality we can assume that the designer will set $\gamma_{A,1} = \gamma_{B,1} = 1$. Since $v > 2$, this shows that there is no equilibrium in which neither contestant puts forth effort and the inequality given in the theorem immediately follows. ■

Lemma 1 *The following relationships between posterior beliefs about the judges bias and the designer's strategy hold when both contestants are conjectured to exert effort in the first contest and* $\gamma_{i,1} > 1/2$ *for* $i = A, B$.

$$\frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{-i,1}} > \frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{i,1}} > 0 \text{ and } \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{i,1}} < \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{-i,1}} < 0.$$

Proof. The posteriors when both contestants are conjectured to put in effort in period one are

$$\begin{aligned} p_{i,2}(\Gamma, i) &= \frac{\gamma_{i,1} p_{i,1}}{\frac{1}{2}\gamma_{i,1}(1 + p_{i,1} - p_{-i,1}) + \frac{1}{2}(1 - \gamma_{-i,1})(1 + p_{-i,1} - p_{i,1})} \\ p_{i,2}(\Gamma, -i) &= \frac{(1 - \gamma_{i,1}) p_{i,1}}{\frac{1}{2}\gamma_{-i,1}(1 + p_{-i,1} - p_{i,1}) + \frac{1}{2}(1 - \gamma_{i,1})(1 + p_{i,1} - p_{-i,1})}. \end{aligned}$$

Taking partial derivatives

$$\begin{aligned} \frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{i,1}} &= \frac{\frac{1}{2}(1 - \gamma_{-i,1})(1 + p_{-i,1} - p_{i,1})}{\left(\frac{1}{2}\gamma_{i,1}(1 + p_{i,1} - p_{-i,1}) + \frac{1}{2}(1 - \gamma_{-i,1})(1 + p_{-i,1} - p_{i,1})\right)^2} \\ \frac{\partial p_{i,2}(\Gamma, i)}{\partial \gamma_{-i,1}} &= \frac{\frac{1}{2}\gamma_{i,1}(1 + p_{-i,1} - p_{i,1})}{\left(\frac{1}{2}\gamma_{i,1}(1 + p_{i,1} - p_{-i,1}) + \frac{1}{2}(1 - \gamma_{-i,1})(1 + p_{-i,1} - p_{i,1})\right)^2} \\ \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{i,1}} &= \frac{-\frac{1}{2}\gamma_{-i,1}(1 + p_{-i,1} - p_{i,1})}{\left(\frac{1}{2}\gamma_{-i,1}(1 + p_{-i,1} - p_{i,1}) + \frac{1}{2}(1 - \gamma_{i,1})(1 + p_{i,1} - p_{-i,1})\right)^2} \\ \frac{\partial p_{i,2}(\Gamma, -i)}{\partial \gamma_{-i,1}} &= \frac{-\frac{1}{2}(1 - \gamma_{i,1})(1 + p_{-i,1} - p_{i,1})}{\left(\frac{1}{2}\gamma_{-i,1}(1 + p_{-i,1} - p_{i,1}) + \frac{1}{2}(1 - \gamma_{i,1})(1 + p_{i,1} - p_{-i,1})\right)^2} \end{aligned}$$

The result follows from $\gamma_{i,1} > 1/2$ and $\gamma_{-i,t} > 1/2$. ■

Lemma 2 *It is optimal for the designer to set* $\gamma_{A,2} = \gamma_{B,2} = 1$ *in the second period.*

Proof. Notice that in the second period, the analysis of whether each contestant will put forth effort is exactly equivalent to the analysis from the single period game in the proof of Theorem 1, except that $p_{A,1}$, $p_{B,1}$, $\gamma_{A,1}$, and $\gamma_{B,1}$ are replaced by $p_{A,2}$, $p_{B,2}$, $\gamma_{A,2}$, and $\gamma_{B,2}$. Thus increasing $\gamma_{A,2}$ and $\gamma_{B,2}$ increases the incentives to put forth effort, and it is optimal to increase both of them all the way to one. ■

Theorem 2 *Suppose that no type of contestant is confident.*

- *If* $D \leq \frac{(1+C_B)(1+C_A)}{1-C_A C_B}$*, then in any PBE both types of contestant put forth effort in both periods and the designer occasionally overturns the judge.*
- *If* $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B$*, then in any PBE only contestants of type* B *put forth effort in the second period conditional on their type losing in the first. The designer sets* $\gamma_{A,1} = 1$*, and* $\gamma_{B,1}$ *to the minimum level possible so that both types put forth effort in the first period.*
- *If* $1 + C_B < D \leq 1 + C_A$*, then in any PBE only contestants of type* A *put forth effort in the second period conditional on their type losing in the first. The designer sets* $\gamma_{B,1} = 1$*, and* $\gamma_{A,1}$ *to the minimum level possible so that both types put forth effort in the first period.*
- *If* $1 + C_A < D \leq 1$*, then in any PBE neither type of contestant puts forth effort in the second period conditional on their type losing in the first. The designer sets* $\gamma_{A,1}$ *and* $\gamma_{B,1}$ *high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*
- *If* $1 < D$*, then in any PBE contestants of type* A *put forth effort in both periods but contestants of type* B *do not put forth effort in either period.*

Proof. The contestants put forth maximal effort in both periods if there exist $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy both (IC-1), (BC-A), and (BC-B). Since both C_A and C_B are negative, this is true when the intersection of the lines defined by (BC-A) and (BC-B) falls below the line defined by (IC-1). This intersection is where

$$\gamma_{A,1} - C_A \gamma_{B,1} = 1$$
$$\gamma_{B,1} - C_B \gamma_{A,1} = 1.$$

This can be solved to find that the intersection is where $\gamma_{A,1} = \frac{1+C_A}{1-C_A C_B}$ and $\gamma_{B,1} = \frac{1+C_B}{1-C_A C_B}$. This intersection satisfies (IC-1) if

$$\frac{1+C_A}{1-C_A C_B} + \frac{1+C_B}{1-C_A C_B} - 1 \geq D,$$

or $\frac{(1+C_A)(1+C_B)}{1-C_A C_B} \geq D$.

Next, we show the conditions under which there exist $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy both (IC-1) and (BC-A). This is only possible when $C_A > -1$, because otherwise the only $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfied (BC-A) would need to sum to less than one, which violates (IC-1) for *any* v. Furthermore, when $C_A > -1$, lowering $\gamma_{A,1}$ while increasing $\gamma_{B,1}$ by the same value relaxes (BC-A) while having no effect on (IC-1), so we can focus on the case where $\gamma_{B,1} = 1$. Checking whether both (IC-1) and (BC-A) can be satisfied simultaneously is equivalent to checking whether there exists a $\gamma_{A,1}$ which satisfies

$$\gamma_{A,1} \geq D$$

and

$$\gamma_{A,1} \leq 1 + C_A.$$

Thus, it is possible to satisfy (IC-1) and (BC-A) simultaneously if $D \leq 1 + C_A$. Because the case for type B is symmetric, it is possible to satisfy (IC-1) and (BC-B) simultaneously if $D \leq 1 + C_B$.

Suppose that the designer chooses $\gamma_{A,1}$ and $\gamma_{B,1}$ such that (IC-1) and (BC-A) are satisfied, but (BC-B) is not satisfied. In this case, contestants put forth a total of two effort in both periods when type B is awarded the prize, but only one effort in the second period when type A is awarded the prize. Thus, the expected discounted sum of effort is

$$2 + \mathbb{P}(B \text{ wins} | a_{A,1} = a_{B,1} = 1)(2\delta) + \mathbb{P}(A \text{ wins} | a_{A,1} = a_{B,1} = 1)(\delta)$$

which can be expanded as

$$\begin{aligned} 2 + &\left(\frac{1}{2}(1 - \gamma_{A,1})(1 + p_{A,1} - p_{B,1}) + \frac{1}{2}\gamma_{B,1}(1 + p_{B,1} - p_{A,1}) \right)(2\delta) \\ &+ \left(\frac{1}{2}\gamma_{A,1}(1 + p_{A,1} - p_{B,1}) + \frac{1}{2}(1 - \gamma_{B,1})(1 + p_{B,1} - p_{A,1}) \right)(\delta). \end{aligned}$$

This simplifies to

$$2 + \left(\frac{3}{2} + \frac{1}{2}p_{A,1} - \frac{1}{2}p_{B,1} \right)\delta - \left(\frac{3}{2} + \frac{1}{2}p_{A,1} - \frac{1}{2}p_{B,1} \right)\gamma_{A,1}\delta + \left(\frac{3}{2} + \frac{1}{2}p_{B,1} - \frac{1}{2}p_{A,1} \right)\gamma_{B,1}\delta.$$

Thus, the expected discounted sum of effort is decreasing in $\gamma_{A,1}$ and increasing in $\gamma_{B,1}$. The optimal values of $\gamma_{A,1}$ and $\gamma_{B,1}$ subject to them satisfying (IC-1) and (BC-A) (but not (BC-B)) are $\gamma_{B,1} = 1$

and $\gamma_{A,1} = D$. For the optimal choice of $\gamma_{A,1}$ and $\gamma_{B,1}$, the expected discounted sum of effort is

$$2 + 3\delta - \left(\frac{3}{2} + \frac{1}{2}p_{A,1} - \frac{1}{2}p_{B,1}\right)\delta D.$$

A symmetric argument shows that when satisfying (IC-1) and (BC-B) (but not (BC-A)), the expected discounted sum of effort is

$$2 + 3\delta - \left(\frac{3}{2} + \frac{1}{2}p_{B,1} - \frac{1}{2}p_{A,1}\right)\delta D.$$

Thus, when $p_{A,1} \geq p_{B,1}$ and $\frac{(1+C_B)(1+C_A)}{1-C_A C_B} < D \leq 1 + C_B \leq 1 + C_A$, it is optimal to set $\gamma_{A,1} = 1$, $\gamma_{B,1} = D$ so that the contestant of type B puts forth effort conditional on losing but the contestant of type A does not. If $1 + C_B \leq D \leq 1 + C_A$, then it is not possible to ensure that the contestant of type B puts forth effort conditional on losing, and it is optimal to set $\gamma_{A,1} = D$ and $\gamma_{B,1} = 1$.

The proof of Theorem 1 demonstrates that there are values of $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy (IC-1) if and only if $D \leq 1$. This implies that the designer can ensure that (IC-1) but not (BC-A) or (BC-B) can be satisfied when $1 + C_B \leq 1 + C_A < D \leq 1$. Because $p_{A,1} \geq p_{B,1}$ implies that $C_A \geq C_B$ when C_A is negative, this exhausts the cases. ■

Theorem 3 *Suppose that contestants of type A are confident but contestants of type B are not confident.*

- *If $D \leq 1 + C_B$, then in any PBE both types of contestant put forth effort in both periods and the designer occasionally overturns the judge.*
- *If $1 + C_B < D \leq 1$, then in any PBE both types of contestant put forth effort in the first period, but only contestants of type A put forth effort in the second period conditional on their type losing in the first. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*
- *If $1 < D$, then in any PBE contestants of type A put forth effort in both periods but contestants of type B do not put forth effort in either period.*

Proof. Type A being confident implies that (BC-A) is satisfied for any feasible values of $\gamma_{A,1}$ and $\gamma_{B,1}$.

Since (BC-A) is satisfied, the contestants put forth maximal effort in both periods if there exist $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy both (IC-1) and (BC-B). In the proof of Theorem 2 we showed that such

$\gamma_{A,1}$ and $\gamma_{B,1}$ exist if and only if $D \leq 1 + C_B$.

The proof of Theorem 1 demonstrates that there are values of $\gamma_{A,1}$ and $\gamma_{B,1}$ which satisfy (IC-1) if and only if $D \leq 1$. This implies that the designer can ensure that (IC-1) but not (BC-B) can be satisfied when $1 + C_B < D \leq 1$. ■

Theorem 4 *Suppose that both types of contestant are confident. Then both types put forth effort in both periods. The designer sets $\gamma_{A,1}$ and $\gamma_{B,1}$ high enough so that neither contestant has the incentive to stop putting forth effort in the first period.*

Proof. Both players being confident implies that (BC-A) and (BC-B) are both satisfied when $\gamma_{A,1} = \gamma_{B,1} = 1$. Furthermore, when type A is confident,

$$2p_{B,1} + \left(\frac{2}{v} - 1\right)(1 + p_{B,1} - p_{A,1}) \leq 0$$

which can be restated as

$$\begin{aligned}\frac{2}{v}\left(\frac{1}{1 + p_{A,1} - p_{B,1}}\right) &\leq \frac{1 - p_{A,1} - p_{B,1}}{1 - (p_{A,1} - p_{B,1})^2} \\ &\leq 1.\end{aligned}$$

which is exactly (IC-1). Thus, for $\gamma_{A,1} = \gamma_{B,1} = 1$, (IC-1), (BC-A), and (BC-B) are all satisfied and both types of contestant put forth effort in both periods. ■